Online Appendix to

“The Partisan Politics of Law Enforcement”

Contents

A Proofs

Proof of Lemma 1. The net benefit of committing crime is given by equation 4 in the main text. This expression is decreasing in θ_i. Hence, in any sorting equilibrium in which the cutpoints are interior, individuals with $\theta_i \leq \theta_c(\alpha)$ commit crime while individuals with $\theta_i > \theta_c(\alpha)$ refrain from doing so. The net benefit of purchasing private protection is given by

$$\theta_i - c + e\theta_i - \theta_i \left(1 - \frac{\gamma}{\pi}\right)$$

This expression is increasing in θ_i. Hence, individuals with $\theta_i > \theta_p(\alpha)$ purchase private protection while individuals with $\theta_i \leq \theta_p(\alpha)$ refrain from doing so.

Using the properties of the uniform distribution as described in the text to express $\frac{\gamma}{\pi}$ and $\mathbb{E}\left[\theta_{i'} \mid \theta_{i'} \notin \lambda_p\right]$ in terms of θ_c and θ_p yields the following two conditions that have to hold for type θ_c to be indifferent between committing crime and not committing crime and type θ_p to be indifferent between purchasing and not purchasing private protection:

$$\frac{\theta_p}{2} - d\theta_c - \alpha s = 0 \tag{1}$$

$$\theta_p \left(1 - \frac{\theta_c}{\theta_p}\right) = \theta_p - c + e\theta_p \tag{2}$$

The cutpoints given in equations (5) and (6) are the unique solution to this system of equations.

Next, I show that $0 < \theta_c(\alpha) < \frac{\overline{\theta}}{2} < \theta_p(\alpha) < \overline{\theta}$ for all $\alpha \in [0,1]$. Since $\theta_c(\alpha)$ is decreasing in α, $\theta_c(1) > 0$ implies that $\theta_c(\alpha) > 0$ for all $\alpha \in [0,1]$. We thus need

$$\theta_c(1) = \frac{c - 2es}{1 + 2de} > 0,$$

which holds because $e < \frac{1}{2}$ and $c > s$. Similarly, since $\theta_p(\alpha)$ is increasing in α, we have $\theta_p(\alpha) < \overline{\theta}$ for all $\alpha \in [0,1]$ if it is true that $\theta_p(1) < \overline{\theta}$, which holds because

$$\theta_p(1) = \frac{2\,(cd+s)}{1+2de} = \overline{\theta}_{min} < \overline{\theta}.$$

Because $d > 1$, $\overline{\theta}_{min} < \overline{\theta}$ also implies $\theta_c(0) < \frac{\overline{\theta}}{2}$. Since $\theta_c(\alpha)$ is decreasing in α, it is hence true that $\theta_c(\alpha) < \frac{\overline{\theta}}{2}$ for all $\alpha \in [0,1]$. Finally, it remains to show that $\theta_p(0) > \frac{\overline{\theta}}{2}$, which implies that $\theta_p(\alpha) > \frac{\overline{\theta}}{2}$ for all $\alpha \in [0,1]$. This is true because

$$\overline{\theta} < \frac{4dc}{2de+1} = \overline{\theta}_{max}.$$

$d > 1$ and $c > s$ imply $\overline{\theta}_{min} < \overline{\theta}_{max}$.

Proof of Lemma 2. The equilibrium probability that an unprotected individual is losing her income to crime is given by

$$\frac{\gamma}{\pi} = \frac{\theta_c(\alpha)}{\theta_p(\alpha)} = \frac{c - 2e\alpha s}{2(cd+\alpha s)}.$$

In equilibrium, the expected return to crime is equal to

$$\mathbb{E}\left[\theta_{i'} \mid \theta_{i'} \leq \theta_p\right] = \frac{\theta_p(\alpha)}{2} = \frac{cd+\alpha s}{1+2de}.$$

Substituting these expressions into the utility function given in equation 2, the indirect utility $u_i(1,0,\theta_c,\theta_p;\alpha)$ of committing crime without purchasing private protection is given by

$$u_i(1,0,\theta_c,\theta_p;\alpha) = \theta_i\left(1 - \frac{c-2e\alpha s}{2(cd+\alpha s)}\right) + \frac{cd+\alpha s}{1+2de} - d\theta_i - \alpha s + (1-\alpha)b.$$

The second derivative of $u_i(1,0,\theta_c,\theta_p;\alpha)$ with respect to α is negative for $\alpha \in [0,1]$, which proves that $u_i(1,0,\theta_c,\theta_p;\alpha)$ is concave in α for $\alpha \in [0,1]$:

$$\frac{\partial^2 u_i(1,0,\theta_c,\theta_p;\alpha)}{\partial \alpha^2} = -\frac{\theta_i s^2 c\,(1+2de)}{(cd+\alpha s)^3} < 0.$$

The preferred budget share $\alpha^*(1,0;\theta_i)$ of an individual who chooses to remain unprotected and to commit crime is hence given by the solution to the following first order condition:

$$\frac{\partial u_i(1,0,\theta_c,\theta_p;\alpha)}{\partial \alpha} = \frac{\theta_i sc\,(1+2de)}{2(cd+\alpha s)^2} - \frac{2sde}{1+2de} - b = 0.$$

This equation has the following two roots:

$$\alpha_1 = \frac{1}{s}\left[-\sqrt{\frac{\theta_i cs(1+2de)^2}{2\,(b+2de(b+s))}} - cd\right]$$
$$\alpha_2 = \frac{1}{s}\left[\sqrt{\frac{\theta_i cs(1+2de)^2}{2\,(b+2de(b+s))}} - cd\right].$$

We have $\alpha_1 < 0$. Since $\alpha \in [0,1]$, we must have $\alpha^*(1,0;\theta_i) = \alpha_2$.

The indirect utility $u_i(0,0,\theta_c,\theta_p;\alpha)$ of not committing crime and not purchasing private protection is given by

$$u_i(0,0,\theta_c,\theta_p;\alpha) = \theta_i\left(1 - \frac{c-2e\alpha s}{2(cd+\alpha s)}\right) + (1-\alpha)b.$$

The second derivative of $u_i(0,0,\theta_c,\theta_p;\alpha)$ with respect to α is negative for $\alpha \in [0,1]$, which proves that $u_i(0,0,\theta_c,\theta_p;\alpha)$ is concave in α for $\alpha \in [0,1]$:

$$\frac{\partial^2 u_i(0,0,\theta_c,\theta_p;\alpha)}{\partial \alpha^2} = -\frac{\theta_i s^2 c\,(1+2de)}{(cd+\alpha s)^3} < 0.$$

The preferred budget share $\alpha^*(0,0;\theta_i)$ of an individual who chooses not to commit crime and to remain unprotected is hence given by the solution to the following first order condition:

$$\frac{\partial u_i(0,0,\theta_c,\theta_p;\alpha)}{\partial \alpha} = \frac{\theta_i sc\,(1+2de)}{2(cd+\alpha s)^2} - b = 0.$$

This equation has the following two roots:

$$\alpha_3 = \frac{1}{s}\left[-\sqrt{\frac{\theta_i cs(1+2de)}{2b}} - cd\right]$$
$$\alpha_4 = \frac{1}{s}\left[\sqrt{\frac{\theta_i cs(1+2de)}{2b}} - cd\right].$$

We have $\alpha_3 < 0$. Since $\alpha \in [0,1]$, we must have $\alpha^*(0,0;\theta_i) = \alpha_4$.

Finally, the indirect utility $u_i(0,1,\theta_c,\theta_p;\alpha)$ of not committing crime and purchasing private protection is given by

$$u_i(0,1,\theta_c,\theta_p,\alpha) = \theta_i - c + e\theta_i + (1-\alpha)b.$$

$u_i(0,1,\theta_c,\theta_p;\alpha)$ is linear and hence concave in α. Moreover, $u_i(0,1,\theta_c,\theta_p,\alpha)$ is decreasing in α, which implies that the preferred budget share $\alpha^*(0,1;\theta_i)$ of an individual who chooses to purchase private protection but not to commit crime will always lie at the corner, i.e., $\alpha^*(0,1;\theta_i) = 0$.

The last part of lemma 2 requires

$$\alpha^*(1,0;\theta_i) \leq \alpha^*(0,0;\theta_i)$$
$$\implies \frac{1}{s}\left[\sqrt{\frac{\theta_i cs(1+2de)^2}{2\,(b+2de(b+s))}} - cd\right] \leq \frac{1}{s}\left[\sqrt{\frac{\theta_i cs(1+2de)}{2b}} - cd\right]$$
$$\implies \frac{(1+2de)}{\sqrt{b+2de(b+s)}} \leq \sqrt{\frac{1+2de}{b}}$$
$$\implies \sqrt{1+2de} \leq \sqrt{1+2de(1+\frac{s}{b})}$$

Since $\frac{s}{b} > 0$, this condition always holds. The inequality is strict with the exception of instances in which both preferred budget shares lie at the corner, i.e. $\alpha^*(1,0;\theta_i) = \alpha^*(0,0;\theta_i) = 0$ or $\alpha^*(1,0;\theta_i) = \alpha^*(0,0;\theta_i) = 1$.

Proof of Lemma 3. Evaluating the integral in equation (7) results in the following expression for welfare

$$W(\alpha) = \frac{(1+e)\overline{\theta}}{2} - c + (1-\alpha)b + \frac{c^2 d(3+4de) + 2\alpha sc + 4\alpha^2 de^2 s^2}{2(1+2de)^2\overline{\theta}}. \tag{3}$$

The second derivative of $W(\alpha)$ w.r.t. α is given by

$$\frac{\partial^2 W(\alpha)}{\partial \alpha^2} = \frac{4de^2 s^2}{(1+2de)^2\overline{\theta}}.$$

It is easy to verify that $\frac{\partial^2 W(\alpha)}{\partial \alpha^2} > 0$ which proves that $W(\alpha)$ is convex in α. As a consequence, the welfare maximizing budget share α_w^* will always lie at the corner. Welfare at $\alpha = 0$ and $\alpha = 1$ is given by

$$W(0) = \frac{(1+e)\overline{\theta}}{2} - c + b + \frac{c^2 d(3+4de)}{2(1+2de)^2\overline{\theta}}$$
$$W(1) = \frac{(1+e)\overline{\theta}}{2} - c + \frac{c^2 d(3+4de) + 2sc + 4de^2 s^2}{2(1+2de)^2\overline{\theta}}$$

$W(1) \geq W(0)$ as long as

$$b \leq \frac{s(c+2de^2 s)}{(1+2de)^2\overline{\theta}} = \overline{b}_W.$$

It follows that $\alpha_w^* = 1$ if $b \leq \overline{b}_W$ and $\alpha_w^* = 0$ if $b > \overline{b}_W$. Finally, it is easy to see from the above expression that $\frac{\partial \overline{b}_W}{\partial \overline{\theta}} < 0$.

Proof of Proposition 1. Evaluating the integrals in equation (10) yields the following expression for the objective function of party L:

$$V_L(\alpha) = \frac{(1-\alpha)b}{2} + \frac{\overline{\theta}(2\alpha s(1+e) + c(2d-1))}{16(cd+\alpha s)} + \frac{d(c-2\alpha se)^2}{2\overline{\theta}(1+2de)^2}.$$

Differentiating this expression twice w.r.t. α gives

$$\frac{\partial^2 V_L(\alpha)}{\partial \alpha^2} = \frac{4s^2 e^2 d}{\overline{\theta}(1+2de)^2} - \frac{\overline{\theta} s^2 c(1+2de)}{8(cd+\alpha s)^3}.$$

$\frac{\partial^2 V_L(\alpha)}{\partial \alpha^2}$ is increasing in α and decreasing in $\overline{\theta}$. The following condition ensures that $\frac{\partial^2 V_L(\alpha)}{\partial \alpha^2} > 0$ for all $\alpha \in [0,1]$:

$$\overline{\theta} < \frac{4\sqrt{2}ed^2c}{(1+2de)^{\frac{3}{2}}}.$$

It is easy to verify that $\overline{\theta}_{max} < \frac{4\sqrt{2}ed^2c}{(1+2de)^{\frac{3}{2}}}$ if $d > \frac{1}{2e}(1+\sqrt{3}) = \underline{d}$. It follows that $\overline{\theta} < \theta_{max}$ and $d > \underline{d}$ are sufficient to ensure that $V_L(\alpha)$ is convex in α for all $\alpha \in [0,1]$. Hence, the budget share α_L^* that maximizes $V_L(\alpha)$ must lie at the corner. $V_L(0)$ and $V_L(1)$ are given by

$$\begin{aligned} V_L(0) &= \frac{b}{2} + \frac{dc^2}{2\overline{\theta}(1+2de)^2} + \frac{\overline{\theta}}{8} - \frac{\overline{\theta}}{16d} \\ V_L(1) &= \frac{d(c-2se)^2}{2\overline{\theta}(1+2de)^2} + \frac{\overline{\theta}(2s(1+e)+c(2d-1))}{16(cd+s)}. \end{aligned}$$

$V_L(1) \geq V_L(0)$ as long as

$$b \leq \frac{\overline{\theta}s(1+2de)}{8d(cd+s)} - \frac{4des(c-se)}{\overline{\theta}(1+2de)^2} = \overline{b}_L.$$

It follows that $\alpha_L^* = 1$ if $b \leq \overline{b}_L$ and $\alpha_L^* = 0$ if $b > \overline{b}_L$.

Evaluating the integrals in equation (11) yields the following expression for the objective function of party R:

$$V_R(\alpha) = \frac{(1-\alpha)b}{2} - c + \frac{\overline{\theta}\,(6\alpha s(1+e)+c+cd(6+8e))}{16(cd+\alpha s)} + \frac{c(cd+\alpha s)}{\overline{\theta}(1+2de)}.$$

The second derivative of $V_R(\alpha)$ w.r.t. α is given by

$$\frac{\partial^2 V_R(\alpha)}{\partial \alpha^2} = \frac{\overline{\theta}s^2c(1+2ed)}{8(cd+\alpha s)^3}.$$

Clearly, $\frac{\partial^2 V_R(\alpha)}{\partial \alpha^2} > 0$ which proves that $V_R(\alpha)$ is convex in α. Hence, the budget share α_R^* that maximizes $V_R(\alpha)$ must lie at the corner. $V_R(0)$ and $V_R(1)$ are given by

$$V_R(0) = -c + \frac{b}{2} + \frac{\overline{\theta}}{16}\left(\frac{1}{d} + 6 + 8e\right) + \frac{c^2 d}{(1+2de)\overline{\theta}}$$
$$V_R(1) = -c + \frac{\overline{\theta}\,(6s(1+e) + c + cd(6+8e))}{16(cd+s)} + \frac{c(cd+s)}{(1+2de)\overline{\theta}}.$$

It is straightforward to verify that $V_R(1) \geq V_R(0)$ as long as

$$b \leq \frac{2cs}{(1+2de)\overline{\theta}} - \frac{\overline{\theta}s(1+2de)}{8d(cd+s)} = \overline{b}_R.$$

It follows that $\alpha_R^* = 1$ if $b \leq \overline{b}_R$ and $\alpha_R^* = 0$ if $b > \overline{b}_R$.

Finally, consider the ordering of $\overline{b}_L$, $\overline{b}_W$ and $\overline{b}_R$. First, because $c > s$, $d > 1$ and $0 < e < \frac{1}{2}$, $\overline{b}_L < \overline{b}_W < \overline{b}_R$ at $\overline{\theta} = \overline{\theta}_{min}$. Second, let us differentiate $\overline{b}_L$, $\overline{b}_W$ and $\overline{b}_R$ w.r.t $\overline{\theta}$:

$$\frac{\partial \overline{b}_L}{\partial \overline{\theta}} = \frac{s(1+2de)}{8d(cd+s)} + \frac{4des(c-se)}{(1+2de)^2\overline{\theta}^2}$$
$$\frac{\partial \overline{b}_W}{\partial \overline{\theta}} = -\frac{s(c+2de^2 s)}{(1+2de)^2\overline{\theta}^2}$$
$$\frac{\partial \overline{b}_R}{\partial \overline{\theta}} = -\frac{2cs}{(1+2de)\overline{\theta}^2} - \frac{s(1+2de)}{8d(cd+s)}.$$

It is easy to see that $\frac{\partial \overline{b}_L}{\partial \overline{\theta}} > 0$ and $\frac{\partial \overline{b}_W}{\partial \overline{\theta}}, \frac{\partial \overline{b}_R}{\partial \overline{\theta}} < 0$. Moreover, $c > s$, $d > 1$ and $0 < e < \frac{1}{2}$ imply that $\frac{\partial \overline{b}_R}{\partial \overline{\theta}} < \frac{\partial \overline{b}_W}{\partial \overline{\theta}}$, i.e., $\overline{b}_R$ decreases more quickly with $\overline{\theta}$ than $\overline{b}_W$. These facts together with the ordering of $\overline{b}_L$, $\overline{b}_W$ and $\overline{b}_R$ at $\overline{\theta}_{min}$ imply that there must be a unique $\overline{\theta} > \overline{\theta}_{min}$ at which $\overline{b}_L$ intersects $\overline{b}_W$ from below. Likewise, there must be a unique $\overline{\theta}' > \overline{\theta}_{min}$ at which $\overline{b}_R$ intersects $\overline{b}_W$ from above. Solving the following two equalities for $\overline{\theta}$

$$\overline{b}_L = \overline{b}_W$$
$$\overline{b}_R = \overline{b}_W$$

reveals that all three curves have a unique intersection at

$$\overline{\theta} = \frac{2\sqrt{2d(cd+s)(2de(2c-es)+c)}}{\sqrt{(1+2de)^3}} = \overline{\theta}_W.$$

Finally, $c > s$, $d > \underline{d}$ and $0 < e < \frac{1}{2}$ are sufficient to ensure that $\overline{\theta}_W$ is real and that $\overline{\theta}_W < \overline{\theta}_{max}$, which completes the proof of the last part of the proposition.

Proof of Proposition 2. The derivative of $\overline{b}_W$ w.r.t. c is given by

$$\frac{\partial \overline{b}_W}{\partial c} = \frac{s}{(1+2de)^2\overline{\theta}}.$$

The derivative of $\overline{b}_R$ w.r.t. c is given by

$$\frac{\partial \overline{b}_R}{\partial c} = \frac{(1+2de)s\overline{\theta}}{8(cd+s)^2} + \frac{2s}{(1+2de)\overline{\theta}}.$$

The derivative of $\overline{b}_L$ w.r.t. c is given by

$$\frac{\partial \overline{b}_L}{\partial c} = -\frac{4des}{(1+2de)^2\overline{\theta}} - \frac{(1+2de)s\overline{\theta}}{8(cd+s)^2}.$$

It is easily verified that $\frac{\partial \overline{b}_W}{\partial c} > 0$, $\frac{\partial \overline{b}_R}{\partial c} > 0$, $\frac{\partial \overline{b}_L}{\partial c} < 0$, and $\frac{\partial \overline{b}_R}{\partial c} > \frac{\partial \overline{b}_W}{\partial c}$. The derivative of $\overline{\theta}_W$ w.r.t c is given by

$$\frac{\partial \overline{\theta}_W}{\partial c} = \frac{\sqrt{2}\sqrt{d}(s+2d(c+4cde+e(2-de)s))}{\sqrt{(1+2de)^3}\sqrt{(cd+s)(c+4cde-2de^2s)}}.$$

Given the parameter restrictions imposed in the model setup, $\frac{\partial \overline{\theta}_W}{\partial c}$ is real and $\frac{\partial \overline{\theta}_W}{\partial c} > 0$.

Proof of Proposition 3. Equation (3) in the proof of lemma 3 shows that social welfare does not depend on b if $\alpha = 1$, i.e. $W(1;b) = W(1)$. Hence,

$$\mathbb{E}\left[W(1) \mid b \leq \overline{b}_R\right] = \mathbb{E}\left[W(1) \mid b \leq \overline{b}_L\right] = W(1).$$

Moreover, because $W(\alpha)$ is linear in b, these conditional expectations are given by the following expressions

$$\mathbb{E}\left[W(0;b) \mid b > \overline{b}_R\right] = W(0; \mathbb{E}\left[b \mid b > \overline{b}_R\right])$$

and

$$\mathbb{E}\left[W(0;b) \mid b > \overline{b}_L\right] = W(0;\mathbb{E}\left[b \mid b > \overline{b}_L\right]).$$

Given the distribution of b, we have

$$\mathbb{E}\left[b \mid b > \overline{b}_R\right] = \begin{cases} \frac{\overline{b}_R+\overline{b}_W+\epsilon}{2} & \text{if } \overline{\theta} \leq \overline{\theta}_W \\ \frac{h\left(\overline{b}_W+\frac{\epsilon}{2}\right)+(1-h)\left(\frac{\overline{b}_W-\overline{b}_R}{\epsilon}\right)\left(\frac{\overline{b}_R+\overline{b}_W}{2}\right)}{h+(1-h)\left(\frac{\overline{b}_W-\overline{b}_R}{\epsilon}\right)} & \text{if } \overline{\theta} > \overline{\theta}_W, \end{cases}$$

$$\mathbb{E}\left[b \mid b > \overline{b}_L\right] = \begin{cases} \frac{h\left(\overline{b}_W+\frac{\epsilon}{2}\right)+(1-h)\left(\frac{\overline{b}_W-\overline{b}_L}{\epsilon}\right)\left(\frac{\overline{b}_L+\overline{b}_W}{2}\right)}{h+(1-h)\left(\frac{\overline{b}_W-\overline{b}_L}{\epsilon}\right)} & \text{if } \overline{\theta} \leq \overline{\theta}_W \\ \frac{\overline{b}_L+\overline{b}_W+\epsilon}{2} & \text{if } \overline{\theta} > \overline{\theta}_W. \end{cases}$$

Plugging the probabilities derived in the text and the expressions for the conditional expectations into equation (15) yields the following expression for expected welfare

$$\Omega = \begin{cases} \frac{1}{2}\left(\epsilon h - 2c + (1+e)\overline{\theta} + \frac{\Delta^2(h-1+r(1-2h)}{\epsilon} + \frac{c^2 d(3+4de)+2s(c+2de^2 s)}{(1+2de)^2\overline{\theta}}\right) & \text{if } \overline{\theta} \leq \overline{\theta}_W \\ \frac{1}{2}\left(\epsilon h - 2c + (1+e)\overline{\theta} + \frac{\Delta^2(-h+r(2h-1)}{\epsilon} + \frac{c^2 d(3+4de)+2s(c+2de^2 s)}{(1+2de)^2\overline{\theta}}\right) & \text{if } \overline{\theta} > \overline{\theta}_W. \end{cases}$$

Taking the derivative of this expression w.r.t. r yields equation (16) in the text. Inspecting this expression yields the result.

Proof of Proposition 4. If $h = \frac{1}{2}$, $\frac{\partial^2\Omega}{\partial r \partial \overline{\theta}} = 0$ and $\frac{\partial^2\Omega}{\partial r \partial c} = 0$, because $\frac{\partial \Omega}{\partial r} = 0$. Moreover, using the expressions for $\overline{b}_k$ in equations (12) or (13) and for $\overline{b}_W$ in equation (8), we can derive the following expression for Δ^2:

$$\Delta^2 = \left(\frac{s\left(2de^2 s - c(1+4de)\right)}{(1+2de)^2\overline{\theta}} + \frac{(1+2de)s\overline{\theta}}{8d(cd+s)}\right)^2.$$

The derivative of Δ^2 w.r.t. $\overline{\theta}$ is given by

$$\frac{\partial \Delta^2}{\partial \overline{\theta}} = \frac{(s+2des)^2\overline{\theta}}{32d^2(cd+s)^2} - \frac{2s^2\left(c+2de(2c-es)\right)^2}{(1+2de)^4\overline{\theta}^3}.$$

$\frac{\partial \Delta^2}{\partial \overline{\theta}} \leq 0$ if $\overline{\theta} \leq \overline{\theta}_W$ and $\frac{\partial \Delta^2}{\partial \overline{\theta}} > 0$ if $\overline{\theta} > \overline{\theta}_W$. It follows that $\frac{\partial^2\Omega}{\partial r \partial \overline{\theta}} \leq 0$ if $h < \frac{1}{2}$ and $\frac{\partial^2\Omega}{\partial r \partial \overline{\theta}} \geq 0$ if $h > \frac{1}{2}$.

The derivative of Δ^2 w.r.t. c is given by

$$\frac{\partial \Delta^2}{\partial c} = -\frac{s^2\left(8(1+4de)(cd+s)^2+(1+2de)^3\overline{\theta}^2\right)\left(-8d(cd+s)\left(c+2de(2c-es)\right)+(1+2de)^3\overline{\theta}^2\right)}{32d(1+2de)^4(cd+s)^3\overline{\theta}^2}.$$

$\frac{\partial \Delta^2}{\partial c} \geq 0$ if $\overline{\theta} \leq \overline{\theta}_W$ and $\frac{\partial \Delta^2}{\partial c} < 0$ if $\overline{\theta} > \overline{\theta}_W$. It follows that $\frac{\partial^2 \Omega}{\partial r \partial c} \geq 0$ if $h < \frac{1}{2}$ and $\frac{\partial^2 \Omega}{\partial r \partial c} \leq 0$ if $h > \frac{1}{2}$.

Proof of Proposition 5. Behavior in the sorting equilibrium is unaffected by the change in citizens' utility function. Evaluating the integrals in equation (7) using the new utility function, yields the following expression for social welfare:

$$W(\alpha) = \frac{c^2 d(3+4de)+2s\alpha(c+2de^2 s\alpha)}{2(1+2de)^2\overline{\theta}} - c + b\sqrt{1-\alpha} + \frac{\overline{\theta}}{2}(1+e).$$

The second derivative of this expression w.r.t. α is given by

$$\frac{\partial^2 W(\alpha)}{\partial \alpha^2} = \frac{4de^2 s^2}{(1+2de)^2\overline{\theta}} - \frac{b\sqrt{1-\alpha}}{4(1-\alpha)^2}.$$

The following restriction on b ensures that $\frac{\partial^2 W(\alpha)}{\partial \alpha^2} < 0$ for all $\alpha \in [0,1]$:

$$b > \frac{16de^2 s^2}{(1+2de)^2\overline{\theta}} = b_1.$$

Evaluating the integrals in equation (10) using the new utility function, yields the following expression for the objective function of party L:

$$V_L(\alpha) = \frac{1}{2}b\sqrt{1-\alpha} + \frac{2s^2\alpha^2 ed\left(c(de-1)+es\alpha\right)}{(1+2de)^2(cd+s\alpha)\overline{\theta}} + \frac{\overline{\theta}\left(c(2d-1)+2s\alpha(1+e)\right)}{16(cd+s\alpha)} + \frac{c^2 d(cd+(1-4de)s\alpha)}{2(1+2de)^2(cd+s\alpha)\overline{\theta}}.$$

The second derivative of this expression w.r.t. α is given by

$$\frac{\partial^2 V_L(\alpha)}{\partial \alpha^2} = \frac{4de^2 s^2}{(1+2de)^2\overline{\theta}} - \frac{c(1+2de)s^2\overline{\theta}}{8(cd+s\alpha)^3} - \frac{b\sqrt{1-\alpha}}{8(1-\alpha)^2}.$$

The following restriction on b is sufficient to ensure that $\frac{\partial^2 V_L(\alpha)}{\partial \alpha^2} < 0$ for all $\alpha \in [0,1]$:

$$b > \frac{32de^2 s^2}{(1+2de)^2\overline{\theta}} = b_2.$$

Evaluating the integrals in equation (11) using the new utility function, yields the following expression for the objective function of party R:

$$V_R(\alpha) = \frac{3s\alpha\overline{\theta}(1+e)}{8(cd+s\alpha)} + \frac{c\overline{\theta}(1+d(6+8e))}{16(cd+s\alpha)} + \frac{c\left(s\alpha(Gs\alpha+2cd)+c^2d^2\right)}{(1+2de)(cd+s\alpha)\overline{\theta}} + \frac{b\sqrt{1-\alpha}}{2} - c.$$

The second derivative of this expression w.r.t. α is given by

$$\frac{\partial^2 V_R(\alpha)}{\partial\alpha^2} = \frac{1}{8}G\left(\frac{b}{(\alpha-1)\sqrt{1-\alpha}} + \frac{c(1+2de)s^2\overline{\theta}}{(cd+s\alpha)^3}\right).$$

The following restriction on b ensures that $\frac{\partial^2 V_R(\alpha)}{\partial\alpha^2} < 0$ for all $\alpha \in [0,1]$:

$$b > \frac{(1+2de)s^2\overline{\theta}}{c^2d^3} = b_3.$$

It is always the case that $b_1 < b_2$. Moreover, assumption 1 guarantees that $b_3 < b_2$. Hence, $b > b_2 = \underline{b}$ is sufficient for $W(\alpha)$, $V_L(\alpha)$ and $V_R(\alpha)$ to be concave in α for $\alpha \in [0,1]$.

The socially optimal share of public spending on the police, α_W^*, is implicitly defined by the following first order condition:

$$\frac{\partial W(\alpha)}{\partial\alpha} = 0. \tag{4}$$

Define this first order condition as a function $G_W(\cdot)$ of α and the parameters of the model. By the implicit function theorem, the derivative of α_W^* with respect to $\overline{\theta}$ is

$$\frac{\partial\alpha_W^*}{\partial\overline{\theta}} = -\frac{\frac{\partial G_W(\cdot)}{\partial\overline{\theta}}}{\frac{\partial G_W(\cdot)}{\partial\alpha}}.$$

From the proof of concavity, we know that $\frac{\partial G_W(\cdot)}{\partial\alpha} < 0$. Hence, $\frac{\partial\alpha_W^*}{\partial\overline{\theta}}$ will have the same sign as $\frac{\partial G_W(\cdot)}{\partial\overline{\theta}}$. We have

$$\frac{\partial G_W(\cdot)}{\partial\overline{\theta}} = -\frac{s(c+4de^2s\alpha)}{\overline{\theta}^2(1+2de)^2} < 0.$$

Hence, $\frac{\partial\alpha_W^*}{\partial\overline{\theta}} \leq 0$. The inequality is weak because the solution may lie at the corner.

The budget share of policing preferred by party L, α_L^*, is implicitly defined by the following first order condition:

$$\frac{\partial V_L(\alpha)}{\partial \alpha} = 0. \tag{5}$$

Again, define this first order condition as a function $G_L(\cdot)$ of α and the parameters of the model. As before, the implicit function theorem together with concavity of the objective function imply that $\frac{\partial \alpha_L^*}{\partial \bar{\theta}}$ will have the same sign as $\frac{\partial G_L(\cdot)}{\partial \bar{\theta}}$. We have

$$\frac{\partial G_L(\cdot)}{\partial \bar{\theta}} = \frac{c(1+2de)s}{16(cd+s\alpha)^2} + \frac{2des(c-2es\alpha)}{\bar{\theta}^2(1+2de)^2} > 0,$$

because we have assumed $e < \frac{1}{2}$ and $c > s$. Hence, $\frac{\partial \alpha_L^*}{\partial \bar{\theta}} \geq 0$. The inequality is again weak because the solution may lie at the corner.

Finally, the budget share of policing preferred by party R, α_R^*, is implicitly defined by the following first order condition:

$$\frac{\partial V_R(\alpha)}{\partial \alpha} = 0. \tag{6}$$

Again, define this first order condition as a function $G_R(\cdot)$ of α and the parameters of the model. As before, the implicit function theorem together with concavity of the objective function imply that $\frac{\partial \alpha_R^*}{\partial \bar{\theta}}$ will have the same sign as $\frac{\partial G_R(\cdot)}{\partial \bar{\theta}}$. We have

$$\frac{\partial G_R(\cdot)}{\partial \bar{\theta}} = \frac{1}{16}c(1+2de)s\left(-\frac{1}{(cd+s\alpha)^2} - \frac{16}{\bar{\theta}^2(1+2de)^2}\right) < 0.$$

Hence, $\frac{\partial \alpha_L^*}{\partial \bar{\theta}} \leq 0$. The inequality is again weak because the solution may lie at the corner.

Finally, we have $\frac{\partial G_W(\cdot)}{\partial \bar{\theta}} > \frac{\partial G_R(\cdot)}{\partial \bar{\theta}}$. Since both of these derivatives are negative, it follows that $|\frac{\partial G_W(\cdot)}{\partial \bar{\theta}}| < |\frac{\partial G_R(\cdot)}{\partial \bar{\theta}}|$. Similarly, it is the case that $\frac{\partial G_W(\cdot)}{\partial \alpha} < \frac{\partial G_R(\cdot)}{\partial \alpha}$ which in turn implies $|\frac{\partial G_W(\cdot)}{\partial \alpha}| > |\frac{\partial G_R(\cdot)}{\partial \alpha}|$. Taken together, these facts imply $\frac{\partial \alpha_R^*}{\partial \bar{\theta}} \leq \frac{\partial \alpha_W^*}{\partial \bar{\theta}}$, where the weak inequality, as before, reflects that solutions can lie at the corner.

Proof of Proposition 6. Voter i of party k will decide to turn out if

$$w_i > u_i(x_i^{c*}, x_i^{p*}, \theta_c, \theta_p; \alpha_{\neg k}) - u_i(x_i^{c*}, x_i^{p*}, \theta_c, \theta_p; \alpha_k).$$

Using the uniform distribution of w_i, the probability that voter i of party k will turn out can be written as

$$\Pr\{z_{ik} = 1\} = \frac{1}{2} + \frac{u_i(x_i^{c*}, x_i^{p*}, \theta_c, \theta_p; \alpha_k) - u_i(x_i^{c*}, x_i^{p*}, \theta_c, \theta_p; \alpha_{\neg k})}{2w}. \tag{7}$$

The utility comparison across party platforms needs to take into account the optimal crime and protection choices under each platform. For the purposes of exposition, I will consider the case where $\alpha_R > \alpha_L$, i.e., the right party proposes to spend more on the police than the left party. It is easy to show that the case in which $\alpha_R \leq \alpha_L$ results in the same expressions for the parties' expected vote shares. As depicted in figure A.1, $\alpha_R > \alpha_L$ implies $\theta_c(\alpha_R) < \theta_c(\alpha_L)$, i.e., the criminal sector would be smaller under the platform of the right party. Similarly, $\theta_p(\alpha_R) > \theta_p(\alpha_L)$, i.e., fewer citizens would buy private protection under the right party's platform.

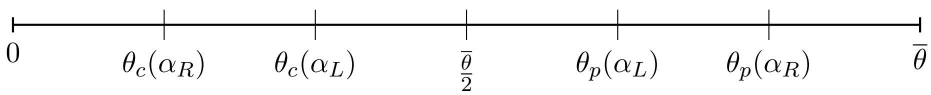

Figure A.1: Ordering of cutpoints for $\alpha_R > \alpha_L$

Hence, among the left party's base, individuals with $\theta_i \in [0, \theta_c(\alpha_R))$ compare the utility from remaining unprotected and committing crime under both platforms. Individuals with $\theta_i \in [\theta_c(\alpha_R), \theta_c(\alpha_L))$ compare the utility of remaining unprotected and abiding by the law under platform α_R to the utility of remaining unprotected but committing crime under α_L. Individuals with $\theta_i \in \left[\theta_c(\alpha_L), \frac{\overline{\theta}}{2}\right)$ compare the utility of remaining unprotected without committing crime under both platforms. The three segments of the right party's base are analogous.

Plugging the relevant utility functions into equation (7) yields expressions for the turnout probability of an individual of a given type. Since individuals are uniformly distributed across the type space, the expected vote shares of the two parties are given by the following integrals:

$$\begin{aligned}\psi_L(\alpha_L,\alpha_R) = &\int_0^{\theta_c(\alpha_R)} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in[0,\theta_c(\alpha_R)]\right\}}{\overline{\theta}}d\theta \\ &+\int_{\theta_c(\alpha_R)}^{\theta_c(\alpha_L)} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in(\theta_c(\alpha_R),\theta_c(\alpha_L)]\right\}}{\overline{\theta}}d\theta \\ &+\int_{\theta_c(\alpha_L)}^{\frac{\overline{\theta}}{2}} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in\left[\theta_c(\alpha_L),\frac{\overline{\theta}}{2}\right]\right\}}{\overline{\theta}}d\theta \end{aligned} \tag{8}$$

$$\begin{aligned}\psi_R(\alpha_L,\alpha_R) = &\int_{\frac{\overline{\theta}}{2}}^{\theta_p(\alpha_L)} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in\left[\frac{\overline{\theta}}{2},\theta_p(\alpha_L)\right]\right\}}{\overline{\theta}}d\theta \\ &+\int_{\theta_p(\alpha_L)}^{\theta_p(\alpha_R)} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in(\theta_p(\alpha_L),\theta_p(\alpha_R)]\right\}}{\overline{\theta}}d\theta \\ &+\int_{\theta_p(\alpha_R)}^{\overline{\theta}} \frac{\Pr\left\{z_{ik}=1 \mid \theta\in\left[\theta_c(\alpha_L),\overline{\theta}\right]\right\}}{\overline{\theta}}d\theta. \end{aligned} \tag{9}$$

Evaluating the integrals in equation (8) yields the following expression for the expected vote share of party L:

$$\psi_L(\alpha_L,\alpha_R) = \frac{1}{4} + \frac{(\alpha_L-\alpha_R)}{\omega}\left(-\frac{b}{4} + \frac{des\left(-c+es(\alpha_L+\alpha_R)\right)}{(1+2de)^2\overline{\theta}} + \frac{c(1+2de)s\overline{\theta}}{32(cd+s\alpha_L)(cd+s\alpha_R)}\right)$$

Differentiating this expression twice w.r.t. α_L gives

$$\frac{\partial^2\psi_L(\alpha_L,\alpha_R)}{\partial\alpha_L^2} = \frac{s^2}{\omega}\left(\frac{2de^2}{(1+2de)^2\overline{\theta}} - \frac{c(1+2de)\overline{\theta}}{16(cd+s\alpha_L)^3}\right).$$

$\frac{\partial^2\psi_L(\alpha_L,\alpha_R)}{\partial\alpha_L^2}$ is increasing in α_L and decreasing in $\overline{\theta}$. The following condition ensures that $\frac{\partial^2\psi_L(\alpha_L,\alpha_R)}{\partial\alpha_L^2} > 0$ for all $\alpha_L \in [0,1]$:

$$\overline{\theta} < \frac{4\sqrt{2}ed^2c}{(1+2de)^{\frac{3}{2}}}.$$

It is easy to verify that $\overline{\theta}_{max} < \frac{4\sqrt{2}ed^2c}{(1+2de)^{\frac{3}{2}}}$ if $d > \frac{1}{2e}(1+\sqrt{3}) = \underline{d}$. It follows that $\overline{\theta} < \theta_{max}$ and $d > \underline{d}$ are sufficient to ensure that $\psi_L(\alpha_L,\alpha_R)$ is convex in α for all $\alpha \in [0,1]$. Hence, the budget share

α_L^* that maximizes $\psi_L(\alpha_L, \alpha_R)$ must lie at the corner. $\psi_L(0, \alpha_R)$ and $\psi_L(1, \alpha_R)$ are given by

$$\begin{aligned}
\psi_L(0, \alpha_R) &= \frac{1}{4} - \frac{\alpha_R}{\omega}\left(-\frac{b}{4} + \frac{des\,(-c + es\alpha_R)}{(1+2de)^2\overline{\theta}} + \frac{(1+2de)s\overline{\theta}}{32d(cd+s\alpha_R)}\right) \\
\psi_L(1, \alpha_R) &= \frac{1}{4} + \frac{(1-\alpha_R)}{\omega}\left(-\frac{b}{4} + \frac{des\,(-c + es(1+\alpha_R))}{(1+2de)^2\overline{\theta}} + \frac{c(1+2de)s\overline{\theta}}{32(cd+s)(cd+s\alpha_R)}\right).
\end{aligned}$$

$\psi_L(1, \alpha_R) \geq \psi_L(0, \alpha_R)$ as long as $b \leq \bar{b}_L$, which establishes the result for party L.

Evaluating the integrals in equation (9) yields the following expression for the expected vote share of party R:

$$\psi_R(\alpha_L, \alpha_R) = \frac{1}{4} + \frac{(\alpha_L - \alpha_R)}{\omega}\left(\frac{b}{4} + \frac{c(1+2de)s\overline{\theta}}{32(cd+s\alpha_L)(cd+s\alpha_R)} - \frac{cs}{2\overline{\theta}(1+2de)}\right).$$

The second derivative of $\psi_R(\alpha_L, \alpha_R)$ w.r.t. α_R is given by

$$\frac{\partial^2 \psi_R(\alpha_L, \alpha_R)}{\partial \alpha_R^2} = \frac{\overline{\theta} s^2 c(1+2ed)}{16\omega(cd+\alpha_R s)^3}.$$

Clearly, $\frac{\partial^2 \psi_R(\alpha_L, \alpha_R)}{\partial \alpha_R^2} > 0$ which proves that $\psi_R(\alpha_L, \alpha_R)$ is convex in α_R. Hence, the budget share α_R^* that maximizes $\psi_R(\alpha_L, \alpha_R)$ must lie at the corner. $\psi_R(\alpha_L, 0)$ and $\psi_R(\alpha_L, 1)$ are given by

$$\begin{aligned}
\psi_R(\alpha_L, 0) &= \frac{1}{4} + \frac{\alpha_L}{\omega}\left(\frac{b}{4} + \frac{(1+2de)s\overline{\theta}}{32d(cd+s\alpha_L)} - \frac{cs}{2\overline{\theta}(1+2de)}\right) \\
\psi_R(\alpha_L, 1) &= \frac{1}{4} + \frac{(\alpha_L - 1)}{\omega}\left(\frac{b}{4} + \frac{c(1+2de)s\overline{\theta}}{32(cd+s)(cd+s\alpha_L)} - \frac{cs}{2\overline{\theta}(1+2de)}\right).
\end{aligned}$$

It is straightforward to verify that $\psi_R(\alpha_L, 1) \geq \psi_R(\alpha_L, 0)$ as long as $b \leq \bar{b}_R$, which establishes the result for party R.

B Additional Information

B.1 Details on Figure 2

Median household income. Calculations are based on small area level (SAL) data from the 2011 census. The census provides data on the number of households within income categories. Median income per small area was calculated as the midpoint of the median category in that area. Small areas are not nested within police precincts. A mapping from police precincts to small areas has been obtained from *OpenUpSA*.[1] The median income per police precinct has been calculated as the weighted average of the small area level income data, where the weights are the share of the precinct's area that belongs to a given small area. The dataset contains close to the universe of police stations (1,140 out of 1,146). Data on the number of officer posts is available for 1,135 of these stations (see below). Outlying observations in the extremely long tail of the income distribution have been removed by top-coding incomes at the 97.5th percentile of the income distribution across these 1,135 police stations. Figure A.2 shows the same plot without top-coding of the income data.

Private security ownership and demand for law enforcement spending. Data stem from the 2016/2017 round of the nationally representative Victims of Crime Survey by StatsSA ($N = 21,095$). The *PSU* number contained in the survey data was used to merge respondents to small areas. A mapping from small areas to police precincts has been obtained from *OpenUpSA*.[2] The mapping indicates which percentage of a given small area falls into which police precinct. Respondents were matched to the precinct that makes up the largest share of the respondent's small area. 221 out of $21,095$ of respondents could not be matched and are excluded from the analysis. In total, at least one respondent has been interviewed in 845 of the police stations. On average, 25 respondents were interviewed in a given station. Of course, the survey was not designed to be representative on the police station level. Hence, results should only be seen as suggestive. The leftmost panel of Figure 2 is based on the following survey question:

- Have you taken any of the following measures to protect yourself against crime and violence?
  - Physical protection measures of home (e.g. burglar doors),

[1] See https://data.openup.org.za/is/dataset/precinct-to-small-area-weights-xrci-yc4x, accessed 11/30/2021.

[2] See https://data.openup.org.za/it/dataset/small-area-to-police-precinct-weights-aurg-34mw, accessed 11/30/2021.

- Physical protection measures of vehicles (e.g. alarm),
- Carrying of weapons (e.g. gun),
- Private security (e.g. paid armed response),
- Self-help groups (e.g. self-defense classes),
- Other

The figure plots the share of respondents per police station who chose “Private security (e.g. paid armed response),” which corresponds most closely to the concept of private security considered in this paper. The middle panel of the figure is based on the following survey question:

- If you could tell the government what to spend money on in order to reduce crime, which ONE of the following would you select?
  - Law enforcement (e.g. more police),
  - The Judiciary/Courts (e.g. harsher penalties for offenders),
  - Social development (e.g. advocacy),
  - Economic development (e.g. job creation)

The figure plots the share of respondents per police station who chose either “Law enforcement (e.g. more police)” or “ The Judiciary/Courts (e.g. harsher penalties for offenders).”

Police personnel. Data on the number of police officer posts (“fixed establishments”) that have been granted to police stations in 2015/2016 have been obtained from the Social Justice Coalition (https://sjc.org.za/), a South African activist group. Fixed establishments are not necessarily reflective of the number of officers actually hired, but of the number of funded positions available. The data set contains information for $1,135$ police stations. The number of posts in each station has been divided by an estimate of the population in each police precinct. Population data have been provided by the Social Justice Coalition but are also based on the 2011 Census. A log transformation has been applied to the data in order to improve presentation.

The line in the plot represents the *gam* smoother as implemented in the R package *ggplot2* and the grey shade represents associated confidence intervals.

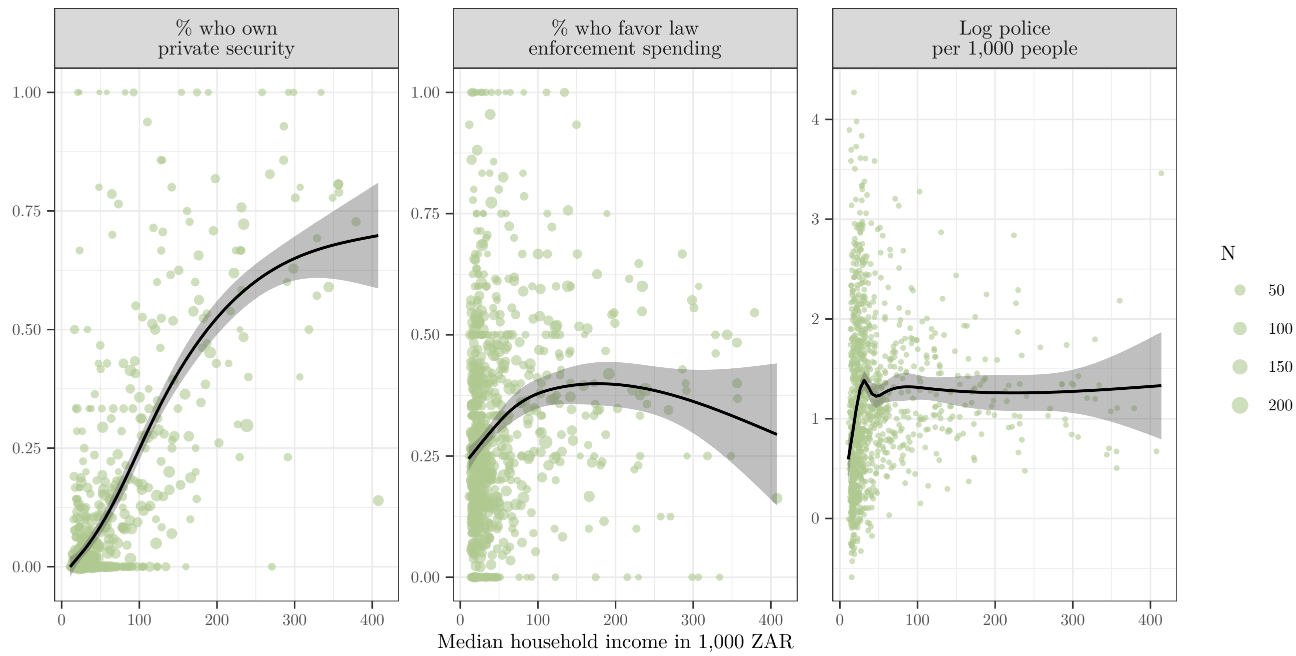

Figure A.2: Private security, demand for law enforcement and police personnel across police stations in South Africa – without top-coding of income

B.2 Details on data from American Housing Survey

The wording of the two survey questions that the main text refers to is as follows:

- Is your community surrounded by walls or fences preventing access by persons other than residents?
- Does access to your community require a special entry system such as entry codes, key cards, or security guard approval?

2009 is the most recent year for which responses to these questions are publicly available. Observations have been weighted using the provided survey weights. The income difference is less stark among renters: $43,117$ USD ($N = 1,397$) among households with and $38,170$ USD ($N = 12,647$) among households without fences or access control. The presence of gating and access control among renters is less likely to capture the kind of private enclaves that debates about the privatization of security focus on (Blakely and Snyder, 1997).

B.3 Details on Figure 3

Data come from the 2016 Cooperative Congressional Election Survey ($N = 64,600$). Respondents who did not respond to a given question have been excluded from the calculation of means and confidence intervals. Observations have been weighted using the provided survey weights. $6,521$ respondents were excluded because they were missing information on family income. The question wording is as follows:

- *Panel 1*: Do you support or oppose increasing the number of police on the street by 10 percent, even if it means fewer funds for other public services?
  - 0 = Oppose
  - 1 = Support
- *Panel 2*: Do you support or oppose increasing prison sentences for felons who have already committed two or more serious or violent crimes?
  - 0 = Oppose
  - 1 = Support
- *Panel 3*: State legislatures must make choices when making spending decisions on important state programs. Would you like your legislature to increase or decrease spending on law enforcement?
- 0 = Greatly decrease
- 1 = Slightly decrease
- 2 = Maintain
- 3 = Slightly increase
- 4 = Greatly increase

References

Blakely, Edward J and Mary Gail Snyder. 1997. *Fortress America: Gated Communities in the United States*. Washington, D.C.: Brookings Institution Press.