Online Appendix to "Federalism and Ideology"

Appendix

A Proofs of Theoretical Results

Proof of Lemma 1. Using the expressions for $U^e_{j,k}(\mathbf{x}^*_t(z_k,\mathbf{c}_t),\mathbf{y})$ and substituting for $y_2 = -y_1$ and $z_R = -z_L$ yields

For executive L:

$$U^e_{L,L}(\mathbf{x}^*_t(z_L,\mathbf{c}_t)) = \begin{cases} y_1^2(-8\gamma(\omega-1)+6\omega-8)-2\omega z_L^2, & \text{if } \mathbf{c}_t=(0,0) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)+(\omega-2)\omega z_L^2, & \text{if } \mathbf{c}_t=(0,1) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)+(\omega-2)\omega z_L^2, & \text{if } \mathbf{c}_t=(1,0) \\ 2(\omega-1)\left(y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)+\omega z_L^2\right), & \text{if } \mathbf{c}_t=(1,1) \end{cases}$$

$$U^e_{L,R}(\mathbf{x}^*_t(-z_L,\mathbf{c}_t)) = \begin{cases} y_1^2(-8\gamma(\omega-1)+6\omega-8)-2\omega z_L^2, & \text{if } \mathbf{c}_t=(0,0) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)-\omega(3\omega+2)z_L^2, & \text{if } \mathbf{c}_t=(0,1) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)-\omega(3\omega+2)z_L^2, & \text{if } \mathbf{c}_t=(1,0) \\ 2(\omega-1)y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)-2\omega(3\omega+1)z_L^2, & \text{if } \mathbf{c}_t=(1,1) \end{cases}$$

For executive R:

$$U^e_{R,R}(\mathbf{x}^*_t(-z_L,\mathbf{c}_t)) = \begin{cases} y_1^2(-8\gamma(\omega-1)+6\omega-8)-2\omega z_L^2, & \text{if } \mathbf{c}_t=(0,0) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)+(\omega-2)\omega z_L^2, & \text{if } \mathbf{c}_t=(0,1) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)+(\omega-2)\omega z_L^2, & \text{if } \mathbf{c}_t=(1,0) \\ 2(\omega-1)\left(y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)+\omega z_L^2\right), & \text{if } \mathbf{c}_t=(1,1) \end{cases}$$

$$U^e_{R,L}(\mathbf{x}^*_t(z_L,\mathbf{c}_t)) = \begin{cases} y_1^2(-8\gamma(\omega-1)+6\omega-8)-2\omega z_L^2, & \text{if } \mathbf{c}_t=(0,0) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)-\omega(3\omega+2)z_L^2, & \text{if } \mathbf{c}_t=(0,1) \\ y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right)+ & \\ \qquad 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)-\omega(3\omega+2)z_L^2, & \text{if } \mathbf{c}_t=(1,0) \\ 2(\omega-1)y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)-2\omega(3\omega+1)z_L^2, & \text{if } \mathbf{c}_t=(1,1) \end{cases}$$

Subtracting expressions for particular executive and centralization profile when executives from different parties are in power implies equation (5).

Proof of Lemma 2. The preference orderings for each type of executive and for cut-offs directly follow from comparison of relevant expressions in the proof of Lemma 1.

Proof of Proposition 1. Denote the difference between the terms that correspond to the age 1 period t executive's utility from a strong executive in period $t+1$ as:

$$\forall j \in \{L,R\}:\ \Delta \equiv U^e_{j,j}(\mathbf{x}^*_t(z_j,(1,1))) - \mathbb{E}_{p_{t+1}}\left[U^e_{j,-j}(\mathbf{x}^*_{t+1}(z_{-j},\mathbf{c}_{t+1}))\right].$$

It is straightforward to see that due to symmetrical ideal points of executives and localities, Δ does not depend on the executive's party. In addition $\Delta > 0$, since $U^e_{j,j}(\mathbf{x}^*_t(z_j,(1,1)))$ is the maximum possible stage utility a party j executive can receive and no lottery over other possible policy choices can bring higher utility.

To see how the electoral environment changes the incentives to adopt different centralization profiles, we take the derivative of $V_L(\mathbf{c}_t, p_t)$ with respect to p_t at each possible profile:

$$\frac{\partial V_L(\mathbf{c}_t, p_t)}{\partial p_t} = \begin{cases} (1-q)\,\Delta & \text{if } \mathbf{c}_t = (0,0) \\ q\,4\omega^2 z_L^2 + (1-q)\,\Delta & \text{if } \mathbf{c}_t = (0,1) \\ q\,4\omega^2 z_L^2 + (1-q)\,\Delta & \text{if } \mathbf{c}_t = (1,0) \\ q\,8\omega^2 z_L^2 + (1-q)\,\Delta & \text{if } \mathbf{c}_t = (1,1). \end{cases} \tag{14}$$

It is clear that $V_L(\mathbf{c}_t, p_t)$ is linear and increasing in p_t, and furthermore

$$\frac{\partial V_L((1,1), p_t)}{\partial p_t} > \frac{\partial V_L((1,0), p_t)}{\partial p_t} = \frac{\partial V_L((0,1), p_t)}{\partial p_t} > \frac{\partial V_L((0,0), p_t)}{\partial p_t} > 0.$$

The corresponding derivatives for a party R executive's objective are identical. Since higher levels of centralization have higher slopes with respect to p_t, centralization must be monotonically increasing in p_t.

The existence of the cut-off values of p_t, $\underline{p}$ and $\overline{p}$, for which there are unique most preferred centralization profile for executive from party j can be proven directly by comparison of expressions for $V_j(\mathbf{c}_t, p_t)$ for each executive across different centralization profiles. The expressions are as follows

$$V_L(\mathbf{c}_t, p_t) =$$

$$\begin{cases} (1-q)(U^e_{L,L}(\mathbf{x}^*_t(z_L,(1,1))) - (1-p_t)\Delta) - \\ \qquad q\left(y_1^2(8\gamma(\omega-1)-6\omega+8)+2\omega z_L^2\right) + y_1^2(-8\gamma(\omega-1)+6\omega-8) - 2\omega z_L^2, \text{ if } \mathbf{c}_t = (0,0) \\ (1-q)(U^e_{L,L}(\mathbf{x}^*_t(z_L,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(\omega z_L^2((4p_t-3)\omega-2) + y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)\right) + \\ \qquad y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2) + (\omega-2)\omega z_L^2, \text{ if } \mathbf{c}_t = (0,1) \\ (1-q)(U^e_{L,L}(\mathbf{x}^*_t(z_L,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(\omega z_L^2((4p_t-3)\omega-2) + y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)\right) + \\ \qquad y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2) + (\omega-2)\omega z_L^2, \text{ if } \mathbf{c}_t = (1,0) \\ (1-q)(U^e_{L,L}(\mathbf{x}^*_t(z_L,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(2\omega z_L^2((4p_t-3)\omega-1) + 2(\omega-1)y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)\right) + \\ \qquad 2(\omega-1)\left(y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)+\omega z_L^2\right), \text{ if } \mathbf{c}_t = (1,1) \end{cases}$$

$$V_R(\mathbf{c}_t, p_t) =$$

$$\begin{cases} (1-q)(U^e_{R,R}(\mathbf{x}^*_t(z_R,(1,1))) - (1-p_t)\Delta) - \\ \qquad q\left(y_1^2(8\gamma(\omega-1)-6\omega+8)+2\omega z_L^2\right) + y_1^2(-8\gamma(\omega-1)+6\omega-8) - 2\omega z_L^2, \text{ if } \mathbf{c}_t = (0,0) \\ (1-q)(U^e_{R,R}(\mathbf{x}^*_t(z_R,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(\omega z_L^2((4p_t-3)\omega-2) + y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2)\right) + \\ \qquad y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(-2\gamma(\omega-1)+\omega-2) + (\omega-2)\omega z_L^2, \text{ if } \mathbf{c}_t = (0,1) \\ (1-q)(U^e_{R,R}(\mathbf{x}^*_t(z_R,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(\omega z_L^2((4p_t-3)\omega-2) + y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2)\right) + \\ \qquad y_1^2\left((\omega-2\gamma(\omega-1))^2+2(\omega-2)\right) + 2\omega y_1 z_L(2\gamma(\omega-1)-\omega+2) + (\omega-2)\omega z_L^2, \text{ if } \mathbf{c}_t = (1,0) \\ (1-q)(U^e_{R,R}(\mathbf{x}^*_t(z_R,(1,1))) - (1-p_t)\Delta) + \\ \qquad q\left(2\omega z_L^2((4p_t-3)\omega-1) + 2(\omega-1)y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)\right) + \\ \qquad 2(\omega-1)\left(y_1^2(4(\gamma-1)\gamma(\omega-1)+\omega)+\omega z_L^2\right), \text{ if } \mathbf{c}_t = (1,1) \end{cases}$$

The resulting cut-offs that define the most preferred centralization profile in infinite horizon game are given in equations (9) and (10) and are the same for both executives for their respective probabilities of re-election. It is straightforward to see that the difference between these cut-offs is

$$\overline{p} - \underline{p} = \frac{1+q}{q} \frac{(2\gamma + 2(1-\gamma)/\omega - 1)\, y_1}{z_L}.$$

Since $z_L < 0$ and $y_1 < 0$, and $2\left[\gamma + (1-\gamma)\frac{1}{\omega})\right] - 1 > 0$, this expression is always positive, and thus $\overline{p} > \underline{p}$.

Proof of Proposition 2. The proof directly follows from solving for $\overline{p} = 0$ and $\underline{p} = 0$ using the expressions in equations (9) and (10).

Solving $\underline{p} = 0$ for z_L produces one negative root that simplifies to:

$$z_p' = -\frac{1 + q + 2\sqrt{q(1+q)}}{1-3q} \left(2\gamma + \frac{2(1-\gamma)}{\omega} - 1\right) y_1. \tag{15}$$

For $z_L < z_p'$, $\underline{p} > 0$; Proposition 1 implies that $(0,0)$ is the preferred profile for $p_t < \underline{p}$. This cut-point exists (i.e., is negative) if and only if $q > \frac{1}{3}$. In addition, it is straightforward to show that $z_p' < y_1$ when $q > \frac{1}{3}$: Executives have to be more polarized than localities for $(0,0)$ to be implementable.

Solving $\overline{p} = 0$ for z_L yields two roots. The first is:

$$z_c = \frac{1 + q - 2\sqrt{q(1+q)}}{1-3q} \left(2\gamma + \frac{2(1-\gamma)}{\omega} - 1\right) y_1. \tag{16}$$

For $z_L > z_c$, $\overline{p} < 0$. Again invoking Proposition 1, for $z_L \geq z_c$, $(1,1)$ is the preferred profile for all p_t. It can be shown that when $q > \frac{1}{3}$, $\overline{p} = 0$ can hold only if $z_L > y_1$.

The second root is:

$$z_p'' = \frac{1 + q + 2\sqrt{q(1+q)}}{1 - 3q} \left(2\gamma + \frac{2(1-\gamma)}{\omega} - 1 \right) y_1. \tag{17}$$

For $z_L < z_p''$, $\overline{p} < 0$; by Proposition Proposition 1, for $z_L < z_p''$, $(1, 1)$ is the preferred profile for all p_t. This cut-point exists (i.e., is negative) if and only if $q < \frac{1}{3}$.

Now define

$$z_p = \begin{cases} z_p' & \text{if } q > \frac{1}{3} \\ z_p'' & \text{if } q < \frac{1}{3} \end{cases}$$

Part (i) of the result follows from the derivation of z_c. Part (ii) follows from the derivations of z_c and z_p. Finally, parts (iii) and (iv) follow from the derivation of z_p.

Proof of Proposition 3. We first provide the condition under which profile $(0, 0)$ dominates profile $(1, 1)$. From (12), it is obvious that $W_{00}(\mathbf{x}^*)$ is constant in z_L and $W_{11}(\mathbf{x}^*)$ is maximized at $z_L = 0$. Evaluating both at $z_L = 0$ yields that $W_{00}(\mathbf{x}^*)$ is always higher than $W_{11}(\mathbf{x}^*)$ if:

$$\gamma > \frac{2 + \omega}{2 + 2\omega}.$$

Next, we provide the condition under which profile $(1, 0)$ dominates profile $(0, 1)$. From (12), it is straightforward to verify that $W_{01}(\mathbf{x}^*)$ and $W_{10}(\mathbf{x}^*)$ are parabolas that are symmetric around $z_L = 0$ and maximized at $y_1(1 - 2\gamma)$ and $-y_1(1 - 2\gamma)$ respectively, but are otherwise identical. Thus $(1, 0)$ dominates profile $(0, 1)$ if and only if $-y_1(1 - 2\gamma) < y_1(1 - 2\gamma)$. Since $y_1 < 0$, this is equivalent to $\gamma > 1/2$.

Now consider three cases. (i) If $\gamma > (2+\omega)/(2+2\omega)$, then $(1, 1)$ is never welfare maximizing and $(1, 0)$ dominates $(0, 1)$. Solving for z_L, the welfare under $(1, 0)$ is higher than under $(0, 0)$

if:

$$\begin{aligned} W_{10}(\mathbf{x}^*) &> W_{00}(\mathbf{x}^*) \\ z_L &\in \left(\frac{y_1(2\gamma(\omega-1)-\omega+2)}{\omega}, \frac{y_1(2\gamma(\omega+1)-\omega-2)}{\omega}\right) \end{aligned} \tag{18}$$

Since $y_1(2\gamma(\omega+1)-\omega-2)/\omega > 0$ and $y_1(2\gamma(\omega-1)-\omega+2)/\omega < 0$, $(1,0)$ is welfare maximizing for $z_L > y_1(2\gamma(\omega-1)-\omega+2)/\omega$ and $(0,0)$ is welfare maximizing otherwise.

(ii) If $\gamma \in (1/2, (2+\omega)/(2+2\omega)]$, then $(0,0)$, $(1,0)$, and $(1,1)$ may all be welfare maximizing. The condition for $W_{10}(\mathbf{x}^*) > W_{00}(\mathbf{x}^*)$ is given by (18). The condition for $W_{11}(\mathbf{x}^*) > W_{10}(\mathbf{x}^*)$ evaluates to:

$$z_L \in \left(\frac{y_1(-2\gamma(\omega+1)+\omega+2)}{\omega}, \frac{y_1(-2\gamma(\omega-1)+\omega-2)}{\omega}\right).$$

Since $y_1(-2\gamma(\omega-1)+\omega-2)/\omega > 0$ and $y_1(-2\gamma(\omega+1)+\omega+2)/\omega < 0$ for these values of γ, $W_{11}(\mathbf{x}^*) > W_{10}(\mathbf{x}^*)$ for $z_L > y_1(-2\gamma(\omega+1)+\omega+2)/\omega$. Observe finally that:

$$\frac{y_1(-2\gamma(\omega+1)+\omega+2)}{\omega} - \frac{y_1(2\gamma(\omega-1)-\omega+2)}{\omega} = \frac{y_1(2-4\gamma)}{\omega} > 0,$$

so the interval of z_L for which $(1,0)$ is welfare maximizing is non-empty.

(iii) If $\gamma \leq 1/2$, the analysis is identical to case (ii), but (since $(0,1)$ dominates $(1,0)$) substituting in profile $(0,1)$ for $(1,0)$.

Proof of Proposition 4. Appendix B shows the transition matrix for equilibrium play for the case in which $0 \leq \underline{p} \leq 1$ and $0 \leq \overline{p} \leq 1$.

To calculate the long run probability of the system being in each of the twenty states, we solve the following system of equations:

$$
\begin{aligned}
\pi_{1Ls} &= \frac{1}{2}(1-q)\left(\pi_{1Rs} + \pi_{2Ls} + \pi_{2Rs} + \pi_{1Rw00} + \pi_{1Rw10} + \pi_{1Rw01} + \pi_{1Rw11}\right. \\
&\quad \left. + \pi_{2Lw00} + \pi_{2Rw00} + \pi_{2Lw10} + \pi_{2Rw10} + \pi_{2Lw01} + \pi_{2Rw01} + \pi_{2Lw11} + \pi_{2Rw11}\right) \\
\pi_{1Rs} &= \frac{1}{2}(1-q)\left(\pi_{1Ls} + \pi_{2Ls} + \pi_{2Rs} + \pi_{1Lw00} + \pi_{1Lw10} + \pi_{1Lw01} + \pi_{1Lw11}\right. \\
&\quad \left. + \pi_{2Lw00} + \pi_{2Rw00} + \pi_{2Lw10} + \pi_{2Rw10} + \pi_{2Lw01} + \pi_{2Rw01} + \pi_{2Lw11} + \pi_{2Rw11}\right) \\
\pi_{2Ls} &= \frac{1}{2}(1-q)\left(\pi_{1Ls} + \pi_{1Lw00} + \pi_{1Lw10} + \pi_{1Lw01} + \pi_{1Lw11}\right) \\
\pi_{2Rs} &= \frac{1}{2}(1-q)\left(\pi_{1Rs} + \pi_{1Rw00} + \pi_{1Rw10} + \pi_{1Rw01} + \pi_{1Rw11}\right) \\
\pi_{1Lw00} &= \pi_{1Rs} q \underline{p}(1 - \frac{\underline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Rw00} + \pi_{2Lw00} + \pi_{2Rw00}\right) \\
\pi_{1Rw00} &= \pi_{1Ls} q \underline{p}(1 - \frac{\underline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Lw00} + \pi_{2Lw00} + \pi_{2Rw00}\right) \\
\pi_{1Lw10} &= \pi_{1Rs} q \left(\overline{p} - \underline{p}\right)(1 - \frac{\underline{p} + \overline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Rw10} + \pi_{2Lw10} + \pi_{2Rw10}\right) \\
\pi_{1Rw10} &= \frac{1}{2} q \left(\pi_{1Lw10} + \pi_{2Lw10} + \pi_{2Rw10}\right) \\
\pi_{1Lw01} &= \frac{1}{2} q \left(\pi_{1Rw01} + \pi_{2Lw01} + \pi_{2Rw01}\right) \\
\pi_{1Rw01} &= \pi_{1Ls} q \left(\overline{p} - \underline{p}\right)(1 - \frac{\underline{p} + \overline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Lw01} + \pi_{2Lw01} + \pi_{2Rw01}\right) \\
\pi_{1Lw11} &= \pi_{1Rs} q (1 - \overline{p})(1 - \frac{1 + \overline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Rw11} + \pi_{2Ls} + \pi_{2Rs} + \pi_{2Lw11} + \pi_{2Rw11}\right) \\
\pi_{1Rw11} &= \pi_{1Ls} q (1 - \overline{p})(1 - \frac{1 + \overline{p}}{2}) + \frac{1}{2} q \left(\pi_{1Lw11} + \pi_{2Ls} + \pi_{2Rs} + \pi_{2Lw11} + \pi_{2Rw11}\right) \\
\pi_{2Lw00} &= \pi_{1Ls} q \frac{\underline{p}^2}{2} + \frac{1}{2} q \pi_{1Lw00} \\
\pi_{2Rw00} &= \pi_{1Rs} q \frac{\underline{p}^2}{2} + \frac{1}{2} q \pi_{1Rw00} \\
\pi_{2Lw10} &= \frac{1}{2} q \pi_{1Lw10} \\
\pi_{2Rw10} &= \pi_{1Rs} q \left(\overline{p} - \underline{p}\right) \frac{\underline{p} + \overline{p}}{2} + \frac{1}{2} q \pi_{1Rw10} \\
\pi_{2Lw01} &= \pi_{1Ls} q \left(\overline{p} - \underline{p}\right) \frac{\underline{p} + \overline{p}}{2} + \frac{1}{2} q \pi_{1Lw01} \\
\pi_{2Rw01} &= \frac{1}{2} q \pi_{1Rw01} \\
\pi_{2Lw11} &= \pi_{1Ls} q (1 - \overline{p}) \frac{1 + \overline{p}}{2} + \frac{1}{2} q \pi_{1Lw11} \\
\pi_{2Rw11} &= \pi_{1Rs} q (1 - \overline{p}) \frac{1 + \overline{p}}{2} + \frac{1}{2} q \pi_{1Rw11},
\end{aligned}
$$

where π_{ajwc} refers to the long-run probability of being in a state characterized by a weak (w) executive of age a ($a \in \{1, 2\}$) from party j ($j \in \{L, R\}$) and by centralization profile c ($c \in$

$\{00, 01, 10, 11\}$), while π_{ajs} denotes the long-run probability of being in a state characterized with a strong (s) executive of age a ($a \in \{1, 2\}$) from party j ($j \in \{L, R\}$).

This system provides a unique solution for the twenty long-run probabilities. These long-run probabilities can be used to calculate the four long-run probabilities of being in state with decentralization (ϕ_{00}), partial centralization (ϕ_{10} and ϕ_{01}) and full centralization (ϕ_{11}) used in equation (13) as follows:

$$\phi_{11} = \pi_{1Lw11} + \pi_{1Rw11} + \pi_{2Lw11} + \pi_{2Rw11} + \pi_{2Ls} + \pi_{2Rs} + (\pi_{1Ls} + \pi_{1Rs})(1 - \overline{p})$$

$$\phi_{00} = \pi_{1Lw00} + \pi_{1Rw00} + \pi_{2Lw00} + \pi_{2Rw00} + (\pi_{1Ls} + \pi_{1Rs})\underline{p}$$

$$\phi_{10} = \pi_{1Lw10} + \pi_{1Rw01} + \pi_{2Lw10} + \pi_{2Rw01}$$

$$\phi_{01} = \pi_{1Lw01} + \pi_{1Rw10} + \pi_{2Lw01} + \pi_{2Rw10} + (\pi_{1Ls} + \pi_{1Rs})(\overline{p} - \underline{p}).$$

Comparing Ω_e to welfare from full centralization, $\Omega_{11} = W_{11}(\mathbf{x}^*)$, partial centralization, $\Omega_{10/01} = \frac{1}{2}\left(W_{10}(\mathbf{x}^*) + W_{01}(\mathbf{x}^*)\right)$ and decentralization, $\Omega_{00} = W_{00}(\mathbf{x}^*)$ we can show that

$$\Omega_{00} > \Omega_{10/01} > \Omega_e > \Omega_{11}.$$

B Transition Matrix for Welfare Calculations

Below we present the transition matrix for equilibrium play for $0 \le \underline{p} \le 1$ and $0 \le \overline{p} \le 1$. For legibility, the first 10 and last 10 columns are presented separately. States with a strong executive of age a from party j are denoted by ajs. States with a weak executive of age a from party j and a centralization profile c are denoted by $ajwc$.

	1Ls	1Rs	2Ls	2Rs	1Lw00	1Rw00	1Lw10	1Rw10	1Lw01	1Rw01
1Ls	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	$q\underline{p}\left(1-\frac{\underline{p}}{2}\right)$	0	0	0	$q(\overline{p}-\underline{p})\left(1-\frac{\underline{p}+\overline{p}}{2}\right)$
1Rs	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}(1-q)$	$q\underline{p}\left(1-\frac{\underline{p}}{2}\right)$	0	$q(\overline{p}-\underline{p})\left(1-\frac{\underline{p}+\overline{p}}{2}\right)$	0	0	0
2Ls	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	0	0
2Rs	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	0	0
1Lw00	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}q$	0	0	0	0
1Rw00	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}q$	0	0	0	0	0
1Lw10	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	$\frac{1}{2}q$	0	0
1Rw10	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}q$	0	0	0
1Lw01	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	$\frac{1}{2}q$
1Rw01	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}(1-q)$	0	0	0	0	$\frac{1}{2}q$	0
1Lw11	0	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	0
1Rw11	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}(1-q)$	0	0	0	0	0	0
2Lw00	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0
2Rw00	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0
2Lw10	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0
2Rw10	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0
2Lw01	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$
2Rw01	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	$\frac{1}{2}q$	$\frac{1}{2}q$
2Lw11	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	0	0
2Rw11	$\frac{1}{2}(1-q)$	$\frac{1}{2}(1-q)$	0	0	0	0	0	0	0	0

	1Lw11	1Rw11	2Lw00	2Rw00	2Lw10	2Rw10	2Lw01	2Rw01	2Lw11	2Rw11
1Ls	0	$q(1-\overline{p})\left(1-\frac{1+\overline{p}}{2}\right)$	$q\frac{\underline{p}^2}{2}$	0	0	0	$q(\overline{p}-\underline{p})\frac{\underline{p}+\overline{p}}{2}$	0	$q(1-\overline{p})\frac{1+\overline{p}}{2}$	0
1Rs	$q(1-\overline{p})\left(1-\frac{1+\overline{p}}{2}\right)$	0	0	$q\frac{\underline{p}^2}{2}$	0	$q(\overline{p}-\underline{p})\frac{\underline{p}+\overline{p}}{2}$	0	0	0	$q(1-\overline{p})\frac{1+\overline{p}}{2}$
2Ls	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0	0	0	0	0
2Rs	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0	0	0	0	0
1Lw00	0	0	$\frac{1}{2}q$	0	0	0	0	0	0	0
1Rw00	0	0	0	$\frac{1}{2}q$	0	0	0	0	0	0
1Lw10	0	0	0	0	$\frac{1}{2}q$	0	0	0	0	0
1Rw10	0	0	0	0	0	$\frac{1}{2}q$	0	0	0	0
1Lw01	0	0	0	0	0	0	$\frac{1}{2}q$	0	0	0
1Rw01	0	0	0	0	0	0	0	$\frac{1}{2}q$	0	0
1Lw11	0	$\frac{1}{2}q$	0	0	0	0	0	0	$\frac{1}{2}q$	0
1Rw11	$\frac{1}{2}q$	0	0	0	0	0	0	0	0	$\frac{1}{2}q$
2Lw00	0	0	0	0	0	0	0	0	0	0
2Rw00	0	0	0	0	0	0	0	0	0	0
2Lw10	0	0	0	0	0	0	0	0	0	0
2Rw10	0	0	0	0	0	0	0	0	0	0
2Lw01	0	0	0	0	0	0	0	0	0	0
2Rw01	0	0	0	0	0	0	0	0	0	0
2Lw11	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0	0	0	0	0
2Rw11	$\frac{1}{2}q$	$\frac{1}{2}q$	0	0	0	0	0	0	0	0